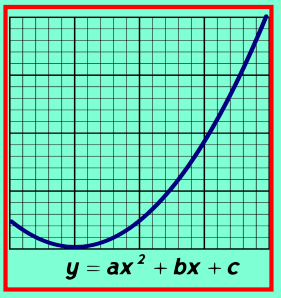
Math 125

Fall 2021

Lecture 45



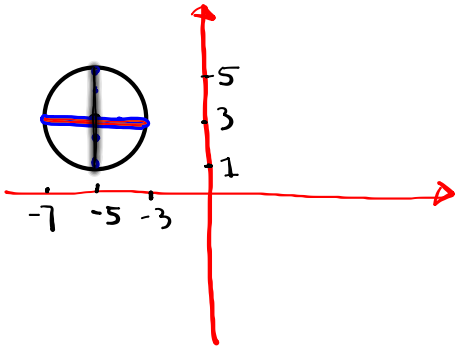
Class QZ 34

Given $(x+5)^2 + (y-3)^2 = 4$

Center $(-5, 3)$

Radius $r = 2$

Draw ✓



Domain $[-7, -3]$ ✓

Range $[1, 5]$ ✓

x-Int None

y-Int.

Variations

1) Directly

y varies **directly** as x $\Rightarrow y = kx$

Constant of Variation

2) Inversely

y varies **inversely** as x $\Rightarrow y = \frac{k}{x}$

Y varies **directly** as x^2. $\Rightarrow y = kx^2$

y is 100 when x is 5. $\Rightarrow 100 = k \cdot 5^2$

$100 = 25k$

$k = 4$

Find y when x is 10.

$y = 4x^2$

$y = 4(10)^2$

$y = 400$

Ex:

y varies directly as square root of x.

$y = k\sqrt{x}$

y is 10 when x is 25.

$10 = k\sqrt{25}$ $\quad 10 = k \cdot 5$ $\quad \boxed{k = 2}$ $\quad y = 2\sqrt{x}$

Find y when x is 100.

$y = 2\sqrt{100} = 2 \cdot 10 = 20$ $\quad \boxed{y = 20}$

Ex:

y varies inversely as cube of x.

$y = \frac{k}{x^3}$

y is 5 when x is 2.

$5 = \frac{k}{2^3}$

$5 = \frac{k}{8}$

$\boxed{k = 40}$

Find y when x is 4.

$y = \frac{40}{x^3}$ $\quad y = \frac{40}{4^3} = \frac{40}{64}$

$\boxed{y = .625}$

y varies inversely as 4th root of x.

$$y = \frac{K}{\sqrt[4]{x}}$$

y is 10 when x is 81.

$$10 = \frac{K}{\sqrt[4]{81}} \qquad 10 = \frac{K}{3} \qquad \boxed{K = 30}$$

Find y when x is 16.

$$y = \frac{K}{\sqrt[4]{x}} = \frac{30}{\sqrt[4]{16}} = \frac{30}{2} \qquad \boxed{y = 15}$$

Joint Variation

Z varies directly as x and inversely as y^2

$$Z = \frac{Kx}{y^2} \qquad 5 = \frac{K \cdot 20}{2^2}$$

Z is 5 when $x = 20$ and $y = 2$.

$$5 = \frac{K \cdot 20}{4}$$

$$5 = 5K$$

$$\boxed{K = 1}$$

Find Z when $x = 40$ and $y = 4$.

$$Z = \frac{x}{y^2} \qquad Z = \frac{40}{4^2} = \frac{40}{16} = \boxed{2.5}$$

Z varies inversely as square root of the sum of x^2 and y^2. $Z = \frac{k}{\sqrt{x^2+y^2}}$

Z is 5 when $x=6$ and $y=8$.

$5 = \frac{k}{\sqrt{6^2+8^2}}$ $\quad 5 = \frac{k}{\sqrt{100}}$ $\quad 5 = \frac{k}{10}$ $\quad \boxed{k=50}$

Find Z when $x=3$ and $y=4$.

$Z = \frac{k}{\sqrt{x^2+y^2}} = \frac{50}{\sqrt{3^2+4^2}} = \frac{50}{\sqrt{25}} = \frac{50}{5} = \boxed{10}$

Z varies directly as square root of the difference of x^2 and y^2.

$Z = k \cdot \sqrt{x^2-y^2}$

Z is 40 when x is 5 and y is 3.

$40 = k \cdot \sqrt{5^2-3^2}$ $\quad 40 = k \cdot \sqrt{16}$ $\quad 40 = k \cdot 4$ $\quad \boxed{k=10}$

Find Z when x is 10 and y is 8.

$Z = k\sqrt{x^2-y^2}$

$Z = 10\sqrt{10^2-8^2} = 10\sqrt{36} = 10 \cdot 6 = \boxed{60}$

Solve

$$\begin{cases} x - y = 2 \\ x^2 + y^2 = 34 \end{cases}$$

$x - y = 2 \Rightarrow x = y + 2$

$(y+2)^2 + y^2 = 34$

$(y+2)(y+2) + y^2 = 34$

$y^2 + 4y + 4 + y^2 = 34$

$2y^2 + 4y + 4 - 34 = 0$

$2y^2 + 4y - 30 = 0$

$y^2 + 2y - 15 = 0$

$(y+5)(y-3) = 0$

$y = -5$ $\quad$ $y = 3$

when $y = -5$

$x = -5 + 2$ $\quad$ $x = -3$ $\Rightarrow (-3, -5)$

when $y = 3$

$x = 3 + 2$ $\quad$ $x = 5$ $\Rightarrow (5, 3)$

$(5, 3), (-3, -5)$

Square-Root Method:

If $x^2 = K$, then $x = \pm\sqrt{K}$

Solve $x^2 = 32$

By S.R.M.

$$x = \pm\sqrt{32} = \pm\sqrt{16}\sqrt{2} = \pm 4\sqrt{2}$$

$$\{\pm 4\sqrt{2}\}$$

Ex:

Solve $x^2 = -100$

By S.R.M.

$x = \pm\sqrt{-100} = \pm\sqrt{100}\sqrt{-1} = \pm 10i$

$\{\pm 10i\}$

Ex: $(2x-1)^2 = 81$

By S.R.M.

$2x-1 = \pm\sqrt{81}$

$2x-1 = \pm 9$

$2x = 1 \pm 9$

$\{-4, 5\}$

$x = \frac{1+9}{2}$

$x = \frac{1+9}{2}$

$x = 5$

$x = \frac{1-9}{2}$

$x = -4$

If $x^2 = K$, then $x = \pm\sqrt{K}$

Solve

$(3x+2)^2 = 10$

By S.R.M.

$3x+2 = \pm\sqrt{10}$

$3x = -2 \pm \sqrt{10}$

$x = \frac{-2 \pm \sqrt{10}}{3}$

If $x^2 = K$, then $x = \pm\sqrt{K}$

$\left\{\frac{-2 \pm \sqrt{10}}{3}\right\}$

Solve

$$(5x+4)^2 = -9$$

By S.R.M.

$$5x+4 = \pm\sqrt{-9}$$

$$5x+4 = \pm\sqrt{9}\sqrt{-1}$$

$$5x+4 = \pm 3i$$

$$5x = -4 \pm 3i$$

$$\boxed{x = \frac{-4}{5} \pm \frac{3}{5}i}$$

$$\left\{\frac{-4}{5} \pm \frac{3}{5}i\right\}$$

Solve

$$x^2 - 8x + 16 = -400$$

$$(x-4)^2 = -400$$

Hint: Express LHS as $(\quad)^2$

By S.R.M.

$$x-4 = \pm\sqrt{-400}$$

$$x = 4 \pm \sqrt{400}\sqrt{-1}$$

$$\boxed{x = 4 \pm 20i} \Rightarrow \{4 \pm 20i\}$$

Solve

$4x^2 + 12x + 9 = -49$

Hint: See last example.

$(2x+3)^2 = -49$

Now use S.R.M.

$2x+3 = \pm\sqrt{-49}$

$2x+3 = \pm 7i$

$2x = -3 \pm 7i$

$x = \frac{-3}{2} \pm \frac{7}{2}i$

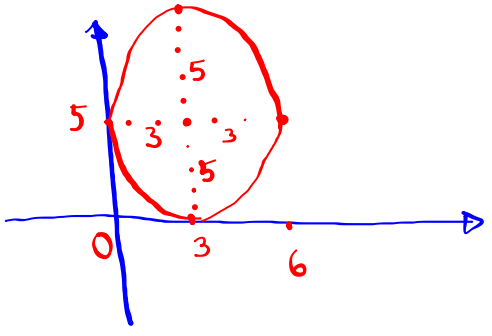
$\left\{\frac{-3}{2} \pm \frac{7}{2}i\right\}$

Class QZ 35

Given $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

Center (3,5)

Draw



Domain & Range

[0,6] [0,10]

All intercepts

x-Int (3,0)

Y-Int (0,5)